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Bouncing brane cosmologies from warped string compactifications

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Abstract: We study the cosmology induced on a brane probing a warped throat region in a Calabi-Yau compactification of type-IIB string theory. For the case of a BPS D3-brane probing the Klebanov-Strassler warped deformed conifold, the cosmology described by a suitable brane observer is a bouncing, spatially flat Friedmann-Robertson-Walker universe with time-varying Newton's constant, which passes smoothly from a contracting to an expanding phase. In the Klebanov-Tseytlin approximation to the Klebanov-Strassler solution the cosmology would end with a big crunch singularity. In this sense, the warped deformed conifold provides a string theory resolution of a spacelike singularity in the brane cosmology. The four-dimensional effective action appropriate for a brane observer is a simple scalar-tensor theory of gravity. In this description of the physics, a bounce is possible because the relevant energy-momentum tensor can classically violate the null energy condition.

Keywords: D-branes, Cosmology of Theories beyond the SM.

Contents

1. Introduction

There has recently been considerable interest in the properties of string theory cosmology. A generic feature of general relativistic cosmologies is the presence of singularities, which is guaranteed under a wide range of circumstances by the singularity theorems [1]. Since string theory has had great success in providing physically sensible descriptions of certain timelike singularities in compactification geometries, one can hope that it will similarly provide insight into the spacelike or null singularities which arise in various cosmologies. Proposals in this direction have appeared in e.g. [2]–[13].

In a slightly different direction, the possibility of localizing models of particle physics on three-branes in a higher-dimensional bulk geometry has motivated a great deal of work on brane-world cosmology (see [14]–[18] and references therein for various examples). Of particular interest to us will be the "mirage" cosmology [14] which is experienced by a D3-brane observer as he falls through a bulk string theory background. In this note, we present a simple and concrete example where such an observer would describe a cosmology which evades the singularity theorems: his universe is a flat FRW model which smoothly interpolates between a collapsing phase and an expanding phase.

The background through which the D3-brane moves is a Klebanov-Strassler (KS) throat region [19] of a IIB Calabi-Yau compactification. Compactifications including such throats, described in [20], yield models with 4d gravity and a warp factor which can vary by many orders of magnitude as one moves in the internal space (as in the proposal of Randall and Sundrum (RS) [21]). The backgrounds discussed in [20] would also admit, in many cases, some number of wandering D3-branes. Such a brane can fall down the KS throat and bounce smoothly back out, as the supergravity background has small curvature everywhere. The induced cosmology on this probe, as described by an observer who holds particle masses *fixed*, is a spatially flat Friedmann-Robertson-Walker universe which begins in a contracting phase, passes smoothly through a minimum scale factor, and then re-expands.[1] A D3-brane probe in this background satisfies a "no-force" condition which makes it possible to control the velocity of the contraction; in addition, the background can be chosen so that the universe is large in Planck units at the bounce. For this reason, the calculations which lead the brane observer to see a bounce are controlled and do not suffer from large stringy or quantum gravity corrections. It is important to note that in this scenario, the effective 4d Newton's constant G_N varies with the scale factor of the universe; this results from the varying overlap of the graviton wavefunction with the D3-brane.

The KS solution is actually a stringy resolution of the singular Klebanov-Tseytlin (KT) supergravity solution [23], which ends with a naked singularity in the infrared. A brane falling into a Klebanov-Tseytlin throat would therefore undergo a singular big crunch. In this sense, the cosmology we study involves a stringy resolution of a spacelike singularity, from the point of view of an observer on the brane.

Although one can describe the cosmological history of these universes using the behavior of the induced metric along the brane trajectory, it is also interesting to consider the 4d effective field theory that a brane resident could use to explain his cosmology. We construct a simple toy model of these cosmologies using a 4d scalar-tensor theory of gravity. The scalar can be identified with the open string scalar field Φ_r (corresponding to radial motion down the warped throat) in the Born-Infeld action for the D3-brane. It is well known that such scalar-tensor theories can classically violate the null energy condition, making a bounce possible. Related facts about scalar field theories coupled to gravity have been exploited previously by Bekenstein and several subsequent authors [24]–[27].

The organization of this note is as follows. In section 2 we use the construction of [20] to study the cosmology on a brane sliding down the KS throat. In section 3 we provide a discussion of the effective scalar-tensor theory of gravity a brane theorist would probably use to explain his observations. We close with some thoughts on further directions in section 4.

Several previous authors have investigated the possibility of bounce cosmologies in scalar-tensor theories and in brane-world models. For FRW models with spherical spatial sections ($k = +1$), examples in various contexts have appeared in [24, 25, 26]. As we were completing this paper, other discussions of bounces in brane-world models appeared in [28, 29]. To the best of our knowledge, this note provides the first controlled example in string theory of a bouncing, spatially flat FRW cosmology with 4d gravity.

2. Brane cosmology in a warped Calabi-Yau compactification

2.1 The compactifications

In [30, 20, 31], warped string compactifications were explored as a means of realizing the scenario of Randall and Sundrum [21] in a string theory context. It was shown that

[1] A different approach to using the KS model to generate an interesting string theory cosmology recently appeared in [22].

compactifications of IIB string theory on Calabi-Yau orientifolds provide the necessary ingredients. In such models, one derives a tadpole condition of the form

$$\frac{1}{4}N_{O3} = N_{D3} + \frac{1}{2(2\pi)^4(\alpha')^2}\int_X H_3 \wedge F_3\,. \tag{2.1}$$

Here X is the Calabi-Yau manifold, N_{O3} and N_{D3} count the number of orientifold planes coming from fixed points of the orientifold action and the number of transverse D3-branes, and H_3, F_3 are the NSNS and RR three-form field strengths of the IIB theory.[2] In general, the left-hand side of (2.1) is nonzero and can be a reasonably large number, giving rise to the possibility of compactifications with large numbers of transverse D3-branes or internal flux quanta. Since both of these lead to nontrivial warping of the metric as a function of the internal coordinates, (2.1) tells us that these Calabi-Yau orientifolds provide a robust setting for finding warped string compactifications [30, 20, 31].

We can make this somewhat vague statement much more precise in the example of the warped deformed conifold. The conifold geometry is defined in $\mathbb{C}^4$ by

$$z_1{}^2 + z_2{}^2 + z_3{}^2 + z_4{}^2 = 0\,. \tag{2.2}$$

It is topologically a cone over $S^2 \times S^3$; we will refer to the direction transverse to the base as the "radial direction" (with small r being close to the tip and large r being far out along the cone). The *deformed* conifold geometry

$$z_1{}^2 + z_2{}^2 + z_3{}^2 + z_4{}^2 = \epsilon^2 \tag{2.3}$$

has two nontrivial 3-cycles, the A-cycle S^3 which collapses as $\epsilon \to 0$, and the dual B-cycle. Klebanov and Strassler found that the infrared region of the geometry which is holographically dual to a cascading $\mathrm{SU}(N+M) \times \mathrm{SU}(N)$ $\mathcal{N}=1$ supersymmetric gauge theory is precisely a warped version of the deformed conifold geometry, with nontrivial 3-form fluxes

$$\frac{1}{(2\pi)^2\alpha'}\int_A F = M\,, \qquad \frac{1}{(2\pi)^2\alpha'}\int_B H = -k \tag{2.4}$$

and $N = kM$. In particular, the space (2.3) is non-singular and the smooth geometry dual to the IR of the gauge theory reflects the confinement of the Yang-Mills theory (with the small parameter ϵ mapping to the exponentially small dynamical scale of the gauge theory). In a cruder approximation to the physics, Klebanov and Tseytlin had earlier found a dual gravity description with a naked singularity [23]; this heuristically corresponds to the unresolved singularity in (2.2).

In [20], the warped, deformed conifold with flux (2.3), (2.4) was embedded in string/F-theory compactifications to 4d. The small r region is as in [19], while at some large r (in the UV of the dual cascading field theory), the solution is glued into a Calabi-Yau manifold. The fluxes give rise to a potential which fixes (many of) the Calabi-Yau moduli (and in particular the ϵ in (2.3)), while the fluxes plus in some cases wandering D3-branes saturate

[2]In an F-theory description, the left-hand side of (2.1) is replaced by $\frac{\chi(X_4)}{24}$, where X_4 is the relevant elliptic Calabi-Yau fourfold.

the tadpole condition (2.1). If one considers one of the cases with $N_{D3} > 0$, then it is natural to imagine a cosmology arising on a wandering D3-brane as it falls down towards the tip of the conifold (2.3).

2.2 The Klebanov-Strassler geometry

The KS metric is given by (we use the conventions of [32])

$$ds^2 = h^{-1/2}(\tau)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(\tau)ds_6^2 \tag{2.5}$$

where ds_6^2 is the metric of the deformed conifold,

$$\begin{aligned} ds_6^2 = \frac{1}{2}\epsilon^{4/3}K(\tau)\Bigg(&\frac{1}{3K^3(\tau)}\left[d\tau^2 + \left(g^5\right)^2\right] + \cosh^2\left(\frac{\tau}{2}\right)\left[\left(g^3\right)^2 + \left(g^4\right)^2\right] + \\ &+ \sinh^2\left(\frac{\tau}{2}\right)\left[\left(g^1\right)^2 + \left(g^2\right)^2\right]\Bigg). \end{aligned} \tag{2.6}$$

Here

$$\begin{aligned} g^1 &= \frac{e^1 - e^3}{\sqrt{2}}, \qquad g^2 = \frac{e^2 - e^4}{\sqrt{2}} \\ g^3 &= \frac{e^1 + e^3}{\sqrt{2}}, \qquad g^4 = \frac{e^2 + e^4}{\sqrt{2}} \\ g^5 &= e^5 \end{aligned} \tag{2.7}$$

where

$$\begin{aligned} e^1 &= -\sin(\theta_1)d\phi_1, \qquad e^2 = d\theta_1 \\ e^3 &= \cos(\psi)\sin(\theta_2)d\phi_2 - \sin(\psi)d\theta_2 \\ e^4 &= \sin(\psi)\sin(\theta_2)d\phi_2 + \cos(\psi)d\theta_2 \\ e^5 &= d\psi + \cos(\theta_1)d\phi_1 + \cos(\theta_2)d\phi_2. \end{aligned} \tag{2.8}$$

ψ is an angular coordinate which ranges from 0 to 4π, while (θ_1, ϕ_1) and (θ_2, ϕ_2) are the conventional coordinates on two S^2s. The function $K(\tau)$ in (2.5) is given by

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3}\sinh(\tau)}. \tag{2.9}$$

Clearly in (2.5) τ plays the role of the "radial" variable in the conifold geometry, with large τ corresponding to large r.

Finally, the function $h(\tau)$ in (2.5) is rather complicated; it is given by the expression

$$h(\tau) = (g_s M\alpha')^2 2^{2/3}\epsilon^{-8/3}I(\tau) \tag{2.10}$$

where

$$I(\tau) = \int_\tau^\infty dx \frac{x\coth(x) - 1}{\sinh^2(x)}(\sinh(2x) - 2x)^{1/3}. \tag{2.11}$$

It will be useful to note that this reaches a maximum at $\tau = 0$ and decreases monotonically as $\tau \to \infty$. There are also nontrivial backgrounds of the NSNS 2-form and RR 2-form potential; their detailed form will not enter here, but they are crucial in understanding why the D3-brane propagates with no force in the background (2.5).

Since the form of $h(\tau)$ will be important in what follows, we take a moment here to give some limits of the behavior of formulae (2.10), (2.11) [32]. For very small τ, one finds $I(\tau) \sim a_0 + O(\tau^2)$, with a_0 a constant of order 1. In this limit the complicated metric (2.5) simplifies greatly (c.f. equation(67) of [32]):

$$\begin{aligned} ds^2 \to & \frac{\epsilon^{4/3}}{2^{1/3} a_0^{1/2} g_s M \alpha'} dx_n dx_n + a_0^{1/2} 6^{-1/3} \left(g_s M \alpha'\right) \times \\ & \times \left(\frac{1}{2} d\tau^2 + \frac{1}{2} \left(g^5\right)^2 + \left(g^3\right)^2 + \left(g^4\right)^2 + \frac{1}{4} \tau^2 \left[\left(g^1\right)^2 + \left(g^2\right)^2 \right] \right). \end{aligned} \tag{2.12}$$

This is $R^{3,1}$ times (the small τ limit of) the deformed conifold. In particular, the S^3 has fixed radius proportional to $\sqrt{g_s M}$, and so the curvature can be made arbitrarily small for large $g_s M$. In the opposite limit of large τ, the metric simplies to Klebanov-Tseytlin form. Introducing the coordinate r via

$$r^2 = \frac{3}{2^{5/3}} \epsilon^{4/3} e^{\frac{2\tau}{3}} \tag{2.13}$$

and using the asymptotic behavior $I(\tau) \sim 3 \times 2^{-1/3} (\tau - \frac{1}{4}) e^{-\frac{4\tau}{3}}$, one finds

$$ds^2 \to \frac{r^2}{L^2 \sqrt{ln(r/r_s)}} dx_n dx_n + \frac{L^2 \sqrt{ln(r/r_s)}}{r^2} dr^2 + L^2 \sqrt{ln(r/r_s)} ds^2_{T^{1,1}} \tag{2.14}$$

where $ds^2_{T^{1,1}}$ is the metric on the Einstein manifold $T^{1,1}$ and $L^2 = \frac{9 g_s M \alpha'}{2\sqrt{2}}$. This means that up to logarithmic corrections, the large τ behavior gives rise to an AdS_5 metric for the x^μ and τ directions. This is the expected behavior from the field theory dual, since large τ corresponds to the UV, where the theory is approximately the Klebanov-Witten $\mathcal{N} = 1$ SCFT [33].

2.3 Trajectory of a falling brane

We will start the D3-brane at some fixed $\tau = \tau^*$ and send it flying towards $\tau = 0$ with a small initial proper velocity v in the radial τ direction. Before describing the trajectory we will briefly explain our notation. τ always indicates the radial coordinate in the KS geometry (2.5) and is dimensionless in our conventions. We will reserve t for proper time (for the infalling brane) and $\dot{}$ for $\frac{d}{dt}$, while ξ represents the coordinate time, in terms of which the metric is

$$ds^2 = h(\tau)^{-\frac{1}{2}} \left(-d\xi^2 + \sum_i dx_i^2 \right) + g_{\tau\tau} d\tau^2 + \text{angles} \tag{2.15}$$

and thus

$$\left(\frac{dt}{d\xi} \right)^2 = h(\tau)^{-\frac{1}{2}} \left(1 - h(\tau)^{\frac{1}{2}} g_{\tau\tau} \left(\frac{d\tau}{d\xi} \right)^2 \right). \tag{2.16}$$

To leading order in the velocity we have $(dt/d\xi)^2 \approx h(\tau)^{-\frac{1}{2}}$.

Proper distance is given by $d = \int d\tau' g_{\tau\tau}^{1/2}$, and proper velocity by $v \equiv \dot{d} = \dot{\tau} g_{\tau\tau}^{1/2}$. The initial values of the position, proper distance, coordinate velocity, and proper velocity are denoted by $\tau_*, d_*, \dot{\tau}_0$ and v_0, respectively.

The D3-brane trajectory is determined by the Born-Infeld action

$$S_{BI} = \frac{-1}{g_s^2 l_s^4} \int d^3\sigma d\xi \left(h(\tau)^{-1} \sqrt{1 - h(\tau)^{\frac{1}{2}} g_{\tau\tau} \left(\frac{d\tau}{d\xi}\right)^2} - h(\tau)^{-1} \right) \tag{2.17}$$

where we have neglected contributions from the U(1) gauge field on the brane. At leading order in a low-velocity expansion, rewritten in terms of derivatives with respect to proper time,

$$S_{BI} = \frac{1}{2g_s^2 l_s^4} \int d^3\sigma d\xi \; h(\tau)^{-1} g_{\tau\tau} \dot{\tau}^2 \tag{2.18}$$

where the cancellation of the potential $h(\tau)^{-1}$ is the realization of the no-force condition. Conservation of energy then yields

$$\dot{\tau}(t)^2 = \dot{\tau}_0^2 \frac{h(t)}{h(\tau_*)} \frac{g_{\tau\tau}(\tau_*)}{g_{\tau\tau}(t)} \tag{2.19}$$

From the profile of $h/g_{\tau\tau}$ it follows that the brane accelerates gradually toward the tip of the conifold. For large τ we may use the KT radial coordinate r (2.13), in terms of which (2.19) is $d^2r/d\xi^2 = 0$, which is another expression of the balancing of gravitational forces and forces due to flux.

2.4 The induced cosmology

An observer on the brane naturally sees an induced metric

$$ds^2_{\text{brane}} = -dt^2 + h^{-1/2}(\tau)\left(dx_1^2 + dx_2^2 + dx_3^2\right). \tag{2.20}$$

But given that the brane trajectory is a function $\tau(t)$, (2.20) gives rise to a standard FRW cosmology

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right) \tag{2.21}$$

with $a(t)$ given by

$$a(t) = h^{-1/4}(\tau(t)). \tag{2.22}$$

Notice that the graviton wavefunction has a τ-dependent overlap with a brane located at various points in the metric (2.5). This is simply the effect exploited in [21]. The dimensionless strength of gravity therefore scales according to

$$G_N(t) m^2_{\text{open}} \sim h(\tau(t))^{-\frac{1}{2}} \sim a(t)^2 \tag{2.23}$$

where m_{open} is the mass of the first oscillating open string mode. A physicist residing on the brane may choose to fix *one* of the dimensionful quantities G_N, m_{open} in order to set his units of length. Grinstein et al. [34] have shown that a brane observer who uses proper distance to measure lengths on the brane will necessarily find fixed masses and variable

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G_N. One can argue for the same system of units by stipulating that elementary particle masses should be used to define the units, and should be considered fixed with time. In this model we will use the mass of the first excited open string mode to fix such a frame; in a more realistic model, one would want other (perhaps "standard model") degrees of freedom to be the relevant massive modes.

A brane observer following an inward-falling trajectory in the background (2.5) would therefore make the following statements.

1. Elementary particle masses, e.g. $m_{\rm open}$, are considered fixed with time.

2. In these units, the proper distance between galaxies on the brane scales with $a(t)$ as in standard FRW cosmology. In consequence, for the infalling brane (moving towards $\tau = 0$) one observes blueshifting of photons.

3. The gravitational coupling on the brane is time-dependent,

$$G_{\rm N}(t) \sim a(t)^2 \,. \tag{2.24}$$

   Therefore, as the universe collapses, the strength of gravity decreases.

In fact, (2.22) together with (2.24) imply that in 4d *Planck* units, the size of the universe remains *fixed*. From this "closed string" perspective, the cosmology is particularly trivial; the brane radial position is described by a scalar field Φ_r in the 4d action which is undergoing some slow time variation (and, for small brane velocity, carries little enough energy that backreaction is not an issue). However, in this frame particle masses vary with time. We find it more natural, as in [34], for a brane observer to view physics in the frame specified by 1-3 above; we will henceforth adopt the viewpoint of such a hypothetical brane cosmologist. In section 3.1 we describe the field redefinition which takes one from the "brane cosmologist" frame to the "closed string" frame in a toy model.

The bounce. As the brane falls from τ^* towards zero, the scale factor decreases monotonically. It hits $\tau = 0$ in finite proper time. However, as is clear from the metric (2.5), there is no real boundary of the space at this point; $\tau = 0$ is analogous to the origin in polar coordinates. The brane smoothly continues back to positive τ, and the scale factor re-expands. Although it is hard to provide an analytical expression for $a(t)$ given the complexity of the expressions (2.10) and (2.11), we can numerically solve for a; a plot appears in figure 1.

In the approximate supergravity dual to the cascading gauge theory studied in [23], there is instead a naked singularity in the region of small τ, which is deformed away by the fluxes (2.4). In the KT approximation to the physics, then, the cosmology on the brane would actually have a spacelike singularity at some finite proper time. The evolution in this background agrees with figure 1 until one gets close to the tip of the conifold; then, in the "unphysical" region of the KT solution, the brane rapidly re-expands, and a singularity of the curvature scalar of the induced metric arises at a finite proper time. A plot of $a(t)$ for this case appears in figure 2.

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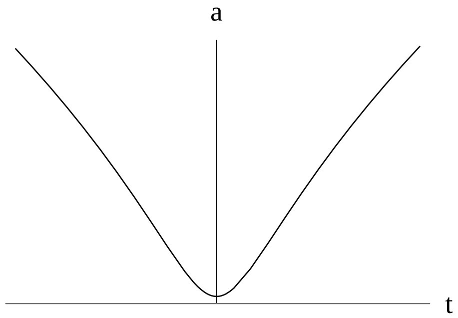

Figure 1: The scale factor $a(t)$ as a function of proper time for a brane near the tip of the Klebanov-Strassler geometry. This particular bounce begins from radial position $\tau = 4$.

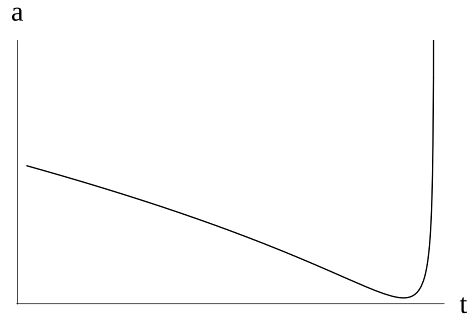

Figure 2: The scale factor $a(t)$ as a function of proper time for a brane near the singularity of the Klebanov-Tseytlin geometry. The explosive growth of $a(t)$ on the right coincides with a curvature singularity in the induced metric.

Hence, we see that string theory in the smooth KS background gives rise to a bouncing brane cosmology, while the KT approximation would have given rise to a cosmology with a spacelike crunch. There has been great success in understanding the resolution of timelike singularities in string theory, so it is heartening to see that in some special cases one can translate those results to learn about spacelike singularities as well.

Limiting behaviors. In the two asymptotic regimes of $\tau \sim 0$ and very large τ, the formulae simplify [32] and the behavior of $a(t)$ can be given explicitly. For small τ, the geometry is just the product (2.12). Hence, in this limit, the brane is effectively falling in an *unwarped* 5d space, and the cosmology is very simple:

$$a(t) = \text{constant} + \mathcal{O}(t^2)\,. \tag{2.25}$$

In the large τ regime, the metric (2.14) differs from AdS_5 by logarithmic corrections, and so the brane trajectory deviates very gradually from that of a D3-brane in AdS. For simplicity we present here the induced cosmology on a D3-brane in AdS; the logarithmic corrections require no new ideas but lead to more complicated formulae. From (2.18), using the D3-brane form of the AdS_5 metric

$$ds^2 = r^2\left(-d\xi^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{dr^2}{r^2} \tag{2.26}$$

we find, in terms of proper time,

$$a^2(t) = a^2(0)\left(1 + 2\frac{\dot{r_0}}{r_0}t\right) \tag{2.27}$$

for a brane with initial position and velocity r_0, $\dot{r_0}$ at $t = 0$. It follows that

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{C}{a^4} \tag{2.28}$$

where $C = a^4(0)(\dot{r_0}/r_0)^2$. Because the right hand side of (2.28) scales like the energy density of radiation, this has been termed "dark radiation" [35, 36]. In the language of [14] it might also be called "mirage matter with equation of state $\rho = 3p$."

The Friedmann equation (2.28) has been thoroughly investigated in the context of Randall-Sundrum models. In particular, just such a law was found to arise on a visible brane which is separated from a Planck brane by an interval whose length varies with time (see [37] and references therein). This is entirely consistent with our scenario, as the Calabi-Yau provides an effective Planck brane and the bulk motion of the probe changes the length of the interval between the branes.

As the brane proceeds to larger τ, eventually it will reach the region where the KS throat has been glued onto a Calabi-Yau space. Beyond that point it is no longer possible for us to say anything universal about the behavior of the brane cosmology.

2.5 Issues of backreaction

There are several issues involving backreaction that merit consideration. To argue that the bounce we have seen in section 2.4 accurately describes the behavior of the brane as it propagates in from τ_* and back out again, we must ensure that the state with nonzero $\dot{\tau}$ on the brane does not contain enough energy to significantly distort the closed string background geometry. In fact we must check both that a motionless brane in the throat creates a negligible backreaction, and that the kinetic energy on the brane does not undergo gravitational collapse (yielding a clumpy brane) on the relevant timescales. It is also important to understand the extent of the backreaction from semiclassical particle production. Finally, the presence of nonzero energy density on the brane leads to a potential for the Calabi-Yau volume modulus (as in section 6 of [38]). We will imagine that this modulus has been fixed and will neglect this effect.

The first concern can be dismissed quickly. In the limit of small g_s the backreaction on the closed string background is small. The second concern needs to be discussed in somewhat more detail. The falling brane necessarily has energy density localized on its worldvolume. After a sufficiently long time this initially uniform energy can become inhomogeneous because of the Jeans instability. In this subsection we demonstrate that, for a suitable choice of the parameters of the KS geometry, this instability is negligible during the bounce portion of the history of the brane universe.

Jeans instability. For a uniform fluid of density ρ, the Jeans instability appears at length scales greater than $L_{\text{Jeans}} \equiv \frac{v_s}{\sqrt{\rho G_N}}$, where v_s is the velocity of sound. Perturbations with this wavelength could destabilize the brane given a time $t_{\text{instability}} \geq L_{\text{Jeans}}$. In terms of the volume V_6 of the Calabi-Yau,

$$G_N = g_s^2 l_s^8 {V_6}^{-1} h(\tau_{UV})^{\frac{1}{2}} h(\tau)^{-\frac{1}{2}} \tag{2.29}$$

where we choose τ_{UV} such that r_{UV} (as given in (2.13)) is of order one (so the throat extends slightly into the KT regime before gluing into the Calabi-Yau). For the compactifications of interest $V_6 \geq l_s^6$,[3] so that for $\tau \leq \tau_{UV}$

$$G_N \leq {g_s}^2 {l_s}^2 \,. \tag{2.30}$$

From (2.18), (2.19), we see that the energy density on the brane is constant,

$$\rho = \frac{1}{2 g_s^2 l_s^4} h(\tau_*)^{-1} g_{\tau\tau}(\tau_*) \dot{\tau_0}^2 \tag{2.31}$$

so

$$t_{\text{instability}} \geq \frac{v_s}{\dot{\tau_0}} h(\tau_*)^{\frac{1}{2}} g_{\tau\tau}(\tau_*)^{-\frac{1}{2}} l_s \,. \tag{2.32}$$

Because the brane accelerates toward the tip of the conifold, to fall from d_* to the tip and rebound requires a time

$$t_{\text{bounce}} \leq \frac{2 d_*}{v_0} \,. \tag{2.33}$$

This leads to (we now drop numerical factors of order one)

$$\frac{t_{\text{bounce}}}{t_{\text{instability}}} < \frac{d_*}{l_s} h(\tau_*)^{-\frac{1}{2}} \,. \tag{2.34}$$

Using the asymptotic form of $I(\tau)$, $K(\tau)$ we find

$$\frac{t_{\text{bounce}}}{t_{\text{instability}}} < \frac{1}{\sqrt{g_s M}} {\tau_*}^{\frac{3}{4}} {l_s}^{-2} \left(\epsilon^2 e^{\tau_*}\right)^{\frac{2}{3}} . \tag{2.35}$$

Because we have glued the KS throat into the Calabi-Yau geometry at a location where $r = r_{UV}$ of (2.13) is of order one, we see that $\epsilon^2 e^{\tau_*} = \mathcal{O}(1)$. This leads to

$$\frac{t_{\text{bounce}}}{t_{\text{instability}}} < \frac{1}{\sqrt{g_s M}} \tau_*^{3/4} \,. \tag{2.36}$$

Finally, since the hierarchy between the UV and IR ends of the throat is exponential in τ_*, it is natural to take τ_* to be a number of order 5–10 (in the language of RS scenarios, τ_* controls the length of the interval in AdS radii, up to factors of π). Therefore, in the supergravity regime where $g_s M \gg 1$, (2.36) demonstrates that we can neglect the Jeans instability on the brane in discussing the dynamics during the bounce.

Particle creation. Because the bounce cosmology is strongly time-dependent, it is also important to consider the spectrum of particles created semiclassically by the bounce. We will argue that the energy density due to such particle production is small enough that its backreaction is negligible.

The bounce geometry (2.21) is conformally trivial, so massless, conformally coupled scalar fields will not be produced by the cosmological evolution. Massive fields break the conformal invariance. The relevant massive scalar fields on the brane are excited

[3] In fact, as discussed in [20], warped compactifications really reproduce the RS scenario when the volume is not very large in string units (since the flux and brane backreaction which produce the warping become larger effects at small Calabi-Yau volume). We are assuming we are at the threshold volume where the warping becomes a significant effect, which should justify the estimate (2.29).

string states with mass $m \geq 1/l_s$. Quite generally we expect that modes with frequencies $\omega \gg \dot{a}/a \equiv H$ will not be significantly populated by the bounce, i.e. the probability that a comoving detector will register such a particle long after the bounce is exponentially small in $\frac{\omega}{H}$. The cases of interest involve slow-moving branes, so the maximum value of H is far below the string scale. Thus we expect the energy density due to particle creation should be quite small.

Concrete calculations of the production of massive scalar and fermion fields in a bouncing $k = 0$ FRW cosmology were carried out in [39] (though the system in consideration there did not satisfy Einstein's equations). The scale factor in [39] has the same limiting behaviors as our own, and the results there are consistent with our expectations. It would be interesting to carry out the relevant particle creation calculation directly in string theory. A particle creation calculation in closed string theory was described in worldsheet (2d conformal field theory) language in [40].

3. Four-dimensional lagrangian description

3.1 Effective lagrangian

In the limit of low matter density on the probe brane, the cosmology is determined entirely by the bulk geometry. The D3-brane trajectory is determined by the Born-Infeld action, and the induced metric along this trajectory provides a time-dependent mirage cosmology. The mirage cosmology proposal of [14] includes another step: one can write down the Friedmann equations for the cosmology and identify the right hand side with mirage density and mirage pressure.

This is not yet an ideal formulation from the perspective of a brane resident. One would like a four-dimensional lagrangian description of the mirage matter, of the cosmological evolution, and of the variation of G_N. In particular, since a bounce in a flat Friedmann-Robertson-Walker universe necessitates violation of the null energy condition, it would be interesting to understand this violation in terms of a 4d lagrangian and energy-momentum tensor. In this section we will propose a toy scalar-tensor lagrangian which admits cosmologies reproducing the basic features of our "bouncing brane" solutions; similar lagrangians have arisen in the study of RS cosmology [41].

The massless fields in our 4d theory include a 4d graviton and the massless open strings on the D3-brane: a U(1) gauge field A_μ, a scalar Φ_r corresponding to radial motion in the compactified throat, and scalars Φ_i, $i = 1, \ldots, 5$ parametrizing motion in the angular coordinates. All other scalar fields are massive. (In fact without a no-force condition there can be a potential and a mass for Φ_r. For simplicity we will work only with the BPS case, but the trajectory of anti-branes in the KS throat would also yield an interesting time-dependent solution.[4]) We will choose to fix the Φ_i, and the requirement of negligible energy density in open string modes on the brane means that A_μ is not relevant for cosmological purposes. This leaves Φ_r and $g_{\mu\nu}$ as the only massless fields entering the 4d lagrangian.

[4] In particular, anti-branes near the tip of the conifold can annihilate by merging with flux [38]. This could potentially lead to a cosmology which begins or ends with a tunneling or annihilation process.

Our goal in this section is to show explicitly how an observer who sees particle masses which depend on Φ_r could change his units of length and see an FRW cosmology with varying G_N. (In section 2.4 we provided several arguments motivating this choice of frame.) Because the full lagrangian for a brane observer in the KS background, including all massive fields, is quite complicated, it will be most practical to work with a simpler lagrangian which has the correct schematic features. In particular, all particle masses depend on Φ_r in the same way, so it will suffice to consider a single massive field χ (which could be, for example, an excited open string mode).

A "mass-varying" lagrangian with the appropriate features is

$$L = \int d^3x\sqrt{-g}\bigg(\frac{R}{16\pi G_N} - \frac{R}{12}\Phi_r^2 - \frac{1}{2}g^{\mu\nu}\nabla_\mu\Phi_r\nabla_\nu\Phi_r - \\ - \frac{1}{2}g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - \frac{1}{2}m^2(\Phi_r)\chi^2 - V(\chi)\bigg) \tag{3.1}$$

where χ is a matter field on the brane whose mass depends on Φ_r as

$$m^2(\Phi_r) \equiv \Omega^2(\Phi_r)\mu^2 \tag{3.2}$$

for fixed μ. The form of the potential for χ and the coupling of χ to the curvature scalar will be unimportant for this analysis, and we will henceforth omit these terms. Note that Φ_r is conformally coupled.

As discussed in section 2.4, an observer confined to the brane most naturally holds fixed the masses of fields on the brane. This can be accomplished by performing the change of variables

$$\tilde{g}_{\mu\nu} = \Omega^2(\Phi_r)g_{\mu\nu} \tag{3.3}$$

$$\tilde{\Phi}_r = \Omega^{-1}(\Phi_r)\Phi_r \tag{3.4}$$

$$\tilde{\chi} = \Omega^{-1}(\Phi_r)\chi \tag{3.5}$$

The resulting "mass-fixed" lagrangian is

$$L = \int d^3x\sqrt{-\tilde{g}}\bigg(\frac{\tilde{R}}{16\pi G_N\Omega^2(\Phi_r)} + \frac{3}{8\pi G_N\Omega(\Phi_r)^4}\tilde{g}^{\mu\nu}\nabla_\mu\Omega\nabla_\nu\Omega - \frac{\tilde{R}}{12}\tilde{\Phi}_r^2 - \\ - \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu\tilde{\Phi}_r\nabla_\nu\tilde{\Phi}_r - \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu\tilde{\chi}\nabla_\nu\tilde{\chi} - \frac{1}{2}\mu^2\tilde{\chi}^2\bigg). \tag{3.6}$$

We have discarded terms which look like $(\nabla\Omega)^2\tilde{\chi}^2$ because $\dot{\Omega} \ll \mu$ (at least in our example, where χ represents a massive string mode). Terms which look like $(\nabla\Omega)^2\tilde{\Phi}_r^2$ cancel due to the conformal coupling of Φ_r.

The effective gravitational coupling is given by

$$G_N{}^{\text{eff}} = G_N\Omega^2(\Phi_r)\,. \tag{3.7}$$

According to the discussion in section 2.4, we expect that $\Omega^2(\Phi_r) = h(\tau(\Phi_r))^{-\frac{1}{2}}$, so indeed the strength of gravity scales as required by (2.24). (We will not need the explicit relation between τ and Φ_r.)

We are interested in the limit where the backreaction due to $\tilde{\Phi}_r, \tilde{\chi}$ is small, so in particular $\tilde{\Phi}_r, \tilde{\chi} \ll m^{\text{eff}}_{Planck}$. This means that for the purpose of solving the Einstein equations in the mass-fixed frame we may neglect terms which are suppressed by a factor of G_N. Defining

$$\gamma = \sqrt{\frac{3}{4\pi G_N}}\Omega^{-1}(\Phi_r) \tag{3.8}$$

we may write the effective lagrangian

$$L = \int d^3x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{12}\gamma^2 + \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu\gamma\nabla_\nu\gamma + \mathcal{O}\left(\frac{\Phi_r}{m_{\text{Planck}}}\right)\right). \tag{3.9}$$

Observe that the kinetic energy term is now negative semidefinite (we are using signature $- + ++$), so it is easy to violate the null energy condition which is relevant (via the singularity theorems) in constraining the behavior of the metric $\tilde{g}_{\mu\nu}$.[5]

The equation of motion which follows from this lagrangian is

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu\gamma - \frac{\tilde{R}}{6}\gamma = \mathcal{O}\left(\frac{\Phi_r}{m_{\text{Planck}}}\right). \tag{3.10}$$

Now let us see that this system reproduces our expectations from section 2.4. Given an FRW cosmology specified by $a(t)$, if we set $\gamma(t) = ca^{-1}(t)$ for some constant c then (3.10) is satisfied identically. From (3.3), (3.4), (3.5) it is clear that we should identify

$$a(t) \propto \Omega(\Phi_r(t)). \tag{3.11}$$

Then the Einstein equations for (3.9) are satisfied if the varying-mass metric $g_{\mu\nu} = \eta_{\mu\nu}$ and the mass-fixed metric $\tilde{g}_{\mu\nu} = a^2(t)\eta_{\mu\nu}$. So as discussed in section 2.4, we have two complementary perspectives: the brane observer uses the mass-fixed action (3.6) and sees an FRW cosmology with varying G_N, while the "closed string" observer sees gravity of fixed strength in Minkowski space.

3.2 Relation to warped backgrounds

We can be slightly more explicit about how the toy model of section 3.1 would be related to a given warped background. Given any function $a(t)$, we can construct a warped background $h(r)$ such that a no-force brane probe of that geometry experiences an induced cosmology specified by $a(t)$. We simply define $\xi = \int \frac{dt}{a(t)}$, $r = v\xi$ (v constant), and $h(r) = a(r)^{-4}$.

A few comments are in order:

1. Very few backgrounds $h(r)$ will correspond to solutions of IIB supergravity. One which does, and indeed corresponds to a D3-brane in the warped deformed conifold, is given by taking $\tau(t)$ to solve (2.19) and setting $a(t) = h^{-\frac{1}{4}}(\tau(t))$ with h given by (2.10).

[5] Notice that because of the non-minimally coupled scalar, it is also possible to violate the null energy condition which governs the behavior of $g_{\mu\nu}$.

2. The no-force condition is only a convenience. We could instead take $r(\xi)$ to be any function of ξ. This would correspond to a brane which accelerates due to external forces. Again, very few systems of this sort arise from known branes of string theory moving in valid supergravity backgrounds.

4. Discussion

As demonstrated in general terms in section 3, and in a special example in string theory in section 2, in the presence of scalar fields it is easy to evade the singularity theorems (from the perspective of a reasonable class of observers), even with a $k = 0$ FRW universe. It therefore seems likely that many examples of such constructions, arising both as cosmologies on D-branes and perhaps even as closed string cosmologies, should be possible. The cosmology we presented is just a slice of evolution between some initial time when we join the brane moving down the throat, and a final time when it is heading into the Calabi-Yau region. The later evolution of our model is then non-universal; it depends on the details of the Calabi-Yau model (or in the language of [21], the detailed structure of the Planck brane). It would be very interesting to write down models with 4d gravity whose dynamics can be controlled for an eternity; some controlled, eternal closed string cosmologies were recently described in [40].

The cosmology discussed here is far from realistic. As a first improvement, one would like to study probe branes with a spectrum of massive fields below the scale $\frac{1}{l_s}$ (which could be called "standard model" fields). It may be possible to construct such examples by using parallel D3-branes which are slightly separated in the radial direction, wrapped Dp-branes with p > 3, or anti-branes in appropriate regimes. It is also important to control the time-variation of G_N during/after nucleosynthesis, since this is highly constrained by experiment (see for instance [42]). To improve the situation, one can envision a program of "cosmological engineering." That is, one could try to design IIB solutions with background fields specifically chosen to give rise to interesting mirage cosmologies (various authors have already proposed mirage models of closed universes [43], inflation with graceful exit [44], asymptotically de Sitter spaces [28], etc., though most of these models do not include 4d gravity). Each desired feature of the cosmology would result in a new condition on the closed string fields. Then one would simply impose these conditions along with the field equations of IIB supergravity.

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